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Programa del Diploma

Markscheme

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Mathematical Studies

Standard level

Paper 2

23 pages

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Paper 2 Markscheme
Instructions to Examiners

Notes: If in doubt about these instructions or any other marking issues, contact your team leader for clarification.

1 Abbreviations

M Marks awarded for **Method**

A Marks awarded for an **Answer** or for **Accuracy**

R Marks awarded for clear **Reasoning**

G Marks awarded for correct solutions obtained from a **Graphic Display Calculator**, when no working shown.

AG **Answer Given** in the question and consequently, marks not awarded.

ft Marks that can be awarded as **follow through** from previous results in the question.

2 Method of Marking

(a) All marking must be done in RM Assessor using the mathematical studies annotations and in accordance with the current document for guidance in e-marking Mathematical Studies SL. It is essential that you read this document before you start marking.

(b) If a question part is completely correct use the number tick annotations to award full marks. If a part is completely wrong use the ***A0*** annotation, otherwise full annotations must be shown.

(c) Working crossed out by the candidate should not be awarded any marks.

(d) Where candidates have written two solutions to a question, only the first solution should be marked.

(e) If correct working results in a correct answer but then further working is developed, indicating a lack of mathematical understanding full marks should **not** be awarded. In most such cases it will be a single final answer mark that is lost. An exception to this may be in numerical answers, where a correct exact value is followed by an incorrect decimal.

Example:

	Correct answer seen	**Further working seen**	**Action**
1.	$8\sqrt{2}$	$5.65685...$ *(incorrect decimal value)*	Award the final ***(A1)*** *(ignore the further working)*
2.	$(x-6)(x+1)$	$x=6$ and -1	Do **not** award the final ***(A1)***

Example: Calculate the gradient of the line passing through the points $(5, 3)$ and $(0, 9)$.

Markscheme	**Candidates' Scripts**	**Marking**
$\frac{9-3}{0-5}$ ***(M1)*** *Award **(M1)** for correct substitution in gradient formula* $=-\frac{6}{5}$ ***(A1)***	(i) $\frac{9-3}{0-5}=-\frac{6}{5}$ Gradient is $=-\frac{6}{5}$ *(There is clear understanding of the gradient.)* $y=-\frac{6}{5}x+9$	***(M1)*** ***(A1)***
	(ii) $\frac{9-3}{0-5}=-\frac{6}{5}$ $y=-\frac{6}{5}x+9$ *(There is confusion about what is required.)*	***(M1)*** ***(A0)***

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3 Follow-through (ft) Marks

Errors made at any step of a solution affect all working that follows. To limit the severity of the penalty, **follow through (ft)** marks can be awarded. Markschemes will indicate where it is appropriate to apply follow through in a question with **'(ft)'**.

(a) Follow through applies only from one part of a question to a subsequent part of the question. Follow through does not apply within the same part.

(b) If an answer resulting from follow through is extremely unrealistic (*eg*, negative distances or incorrect by large order of magnitude) then the final ***A*** mark should not be awarded.

(c) If a question is transformed by an error into a **different**, **much simpler question** then follow through may not apply.

(d) To award follow through marks for a question part, **there must be working present for that part**. An isolated follow through answer, without working is regarded as incorrect and receives no marks **even if it is approximately correct**.

(e) The exception to the above would be in a question which is testing the candidate's use of the GDC, where working will not be expected. **The markscheme will clearly indicate where this applies.**

(f) Inadvertent use of radians will be penalized the first time it occurs. The markscheme will give clear instructions to ensure that only one mark per paper can be lost for the use of radians.

Example: Finding angles and lengths using trigonometry

Markscheme	Candidates' Scripts	Marking
(a) $\frac{\sin A}{3} = \frac{\sin 30}{4}$ ***(M1)(A1)*** *Award **(M1)** for substitution in sine rule formula, **(A1)** for correct substitutions.*	(a) $\frac{\sin A}{4} = \frac{\sin 30}{3}$	***(M1)(A0)*** *(use of sine rule but with wrong values)*
$A = 22.0^\circ\ (22.0243\ldots)$ ***(A1)(G2)***	$A = 41.8^\circ$	***(A0)*** *(**Note:** the 2nd **(A1)** here was not marked **(ft)** and cannot be awarded because there was an earlier error in the **same** question part.)*
(b) $x = 7\tan(22.0243\ldots^\circ)$ ***(M1)*** $= 2.83\ (2.83163\ldots)$ ***(A1)*(ft)**	(b) case (i) $x = 7\tan 41.8^\circ$ $= 6.26$ ***but*** case (ii) 6.26	***(M1)*** ***(A1)*(ft)** ***(G0)*** *since no working shown*

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4 Using the Markscheme

(a) ***A*** marks are **dependent** on the preceding ***M*** mark being awarded, it is **not** possible to award ***(M0)(A1)***. Once an ***(M0)*** has been awarded, all subsequent ***A*** marks are lost in that part of the question, even if calculations are performed correctly, until the next ***M*** mark.

The only exception to this will be for an answer where the accuracy is specified in the question – see section 5.

(b) ***A*** marks are **dependent** on the ***R*** mark being awarded, it is **not** possible to award ***(A1)(R0)***. Hence the ***(A1)*** cannot be awarded for an answer which is correct when no reason or the wrong reason is given.

(c) In paper 2 candidates are expected to demonstrate their ability to communicate mathematics using appropriate working. Answers which are correct but not supported by adequate working will **not always receive full marks**, these unsupported answers are designated ***G*** in the mark scheme as an alternative to the full marks. Example ***(M1)(A1)(A1)(G2)***.

Example: Using trigonometry to calculate an angle in a triangle.

Markscheme	Candidates' Scripts	Marking
(a) $\frac{\sin A}{3}=\frac{\sin 30}{4}$ ***(M1)(A1)*** *Award **(M1)** for substitution in sine rule formula, **(A1)** for correct substitutions.*	(i) $\frac{\sin A}{3}=\frac{\sin 30}{4}$ $A=22.0^{\circ}$	***(M1)(A1)*** ***(A1)***
$A=22.0^{\circ}\ (22.0243\ldots)$ ***(A1)(G2)***	(ii) $A=22.0^{\circ}$ ***Note: G** marks are used only if no working has been shown and the answer is correct.*	***(G2)***

(d) **Alternative methods** may not always be included. Thus, if an answer is wrong then the working must be carefully analysed in order that marks are awarded for a different method consistent with the markscheme.
Where alternative methods for complete questions are included in the markscheme, they are indicated by '**OR**' *etc*.

(e) Unless the question specifies otherwise, accept **equivalent forms**. For example: $\frac{\sin\theta}{\cos\theta}$ for $\tan\theta$.
On the markscheme, these equivalent numerical or algebraic forms will sometimes be written in brackets after the required answer.
Where numerical answers are required as the final answer to a part of a question in the markscheme, the scheme will show, in order:
the 3 significant figure answer worked through from full calculator display;
the exact value $\left(\text{for example } \frac{2}{3} \text{ if applicable}\right)$;
the full calculator display in the form $2.83163\ldots$ as in the example above.
Where answers are given to 3 significant figures and are then used in subsequent parts of the question leading to a **different** 3 significant figure answer, these solutions will also be given.

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(f) As this is an international examination, all valid **alternative forms of notation** should be accepted. Some examples of these are:

Decimal points: 1.7; 1’7; $1 \cdot 7$; 1,7 .

Decimal numbers less than 1 may be written with or without a leading zero: 0.49 or .49 .

Different descriptions of an interval: $3 < x < 5$; (3, 5);] 3, 5 [.

Different forms of notation for set properties (*eg,* complement): A'; $\overline{A}$; A^c; $U - A$;$(A$;$U \setminus A$.

Different forms of logic notation: $\neg p$; p' ; $\tilde{p}$; $\overline{p}$; $\sim p$.

$p \Rightarrow q$; $p \rightarrow q$; $q \Leftarrow p$.

Significance level may be written as α .

(g) Discretionary marks: There will be very rare occasions where the markscheme does not cover the work seen. In such cases the annotation DM should be used to indicate where an examiner has used discretion. Discretion should be used sparingly and if there is doubt and exception should be raised through RM Assessor to the team leader.

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As with previous sessions there will be no whole paper penalty marks for accuracy AP, financial accuracy FP and units UP. Instead these skills will be assessed in particular questions and the marks applied according to the rules given in sections 5, 6 and 7 below.

5 Accuracy of Answers

Incorrect accuracy should be penalized once only in each question according to the rules below.

Unless otherwise stated in the question, all numerical answers should be given exactly or correct to 3 significant figures.

1. If the **candidate's answer** is seen to 4 sf or greater **and** would round to the required 3 sf answer, then award ***(A1)*** and ignore subsequent rounding.

2. If the candidate's unrounded answer is **not** seen then award ***(A1)*** if the answer given is **correctly** rounded to 2 or more significant figures, otherwise ***(A0)***.

Note: If the candidate's unrounded answer is **not** seen and the answer is given correct to 1 sf (correct or not), the answer will be considered wrong and will not count as incorrect accuracy. If this answer is used in subsequent parts, then working must be shown for further marks to be awarded.

3. If a correct 2 sf answer is used in subsequent parts, then working **must** be shown for further marks to be awarded. (This treatment is the same as for following through from an incorrect answer.)

These 3 points (see numbers in superscript) have been summarized in the table below and illustrated in the examples following.

	If candidates final answer is given …					
	Exact or to 4 or more sf (and would **round to the correct 3 sf**)	**Correct to 3 sf**	**Incorrect to 3 sf**	Correct to 2 sf [3]	Incorrect to 2 sf	Correct or incorrect to 1 sf
Unrounded answer seen[1]	Award the final ***(A1)*** irrespective of correct or incorrect rounding					
Unrounded answer not seen[2]	***(A1)***	***(A1)***	***(A0)***	***(A1)***	***(A0)***	***(A0)***
Treatment of subsequent parts	As per MS		Treat as follow through, only if working is seen.[3]			

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Examples:

Markscheme	Candidates' Scripts	Marking
9.43 (9.43398…) ***(A1)***	(i) 9.43398… is seen followed by 9; 9.4; 9.43; 9.434 *etc.* (correctly rounded)	***(A1)***
	(ii) 9.43398… is seen followed by 9.433; 9.44 *etc.* (incorrectly rounded)	***(A1)***
	(iii) 9.4	***(A1)***
	(iv) 9	***(A0)*** *(correct to 1 sf)*
	(v) 9.3	***(A0)*** *(incorrectly rounded to 2 sf)*
	(vi) 9.44	***(A0)*** *(incorrectly rounded to 3 sf)*

Markscheme	Candidates' Scripts	Marking
7.44 (7.43798…) ***(A1)***	(i) 7.43798… is seen followed by 7; 7.4; 7.44; 7.438 *etc.* (correctly rounded)	***(A1)***
	(ii) 7.43798… is seen followed by 7.437; 7.43 *etc.* (incorrectly rounded)	***(A1)***
	(iii) 7.4	***(A1)***
	(iv) 7	***(A0)*** *(correct to 1 sf)*
	(v) 7.5	***(A0)*** *(incorrectly rounded to 2 sf)*
	(vi) 7.43	***(A0)*** *(incorrectly rounded to 3 sf)*

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Example: ABC is a right angled triangle with angle $ABC = 90^\circ$, $AC = 32\text{ cm}$ and $AB = 30\text{ cm}$. Find (a) the length of BC, (b) The area of triangle ABC.

Markscheme	Candidates' Scripts	Marking
(a) $BC = \sqrt{32^2 - 30^2}$ ***(M1)*** *Award **(M1)** for correct substitution in Pythagoras' formula* $= 11.1\ \left(\sqrt{124}, 11.1355...\right)(\text{cm})$ ***(A1)*** (b) $\text{Area} = \frac{1}{2} \times 30 \times 11.1355...$ ***(M1)*** *Award **(M1)** for correct substitution in area of triangle formula* $= 167(167.032...)\,(\text{cm}^2)$ ***(A1)*(ft)**	(a) $BC = \sqrt{32^2 - 30^2}$ $11\,(\text{cm})$ *(2 sf answer only seen, but correct)* (b) case (i) $\text{Area} = \frac{1}{2} \times 30 \times 11$ *(working shown)* $= 165\,(\text{cm}^2)$ case (ii) $= 165\,(\text{cm}^2)$ *(No working shown, the answer 11 is treated as a ft, so no marks awarded here)*	***(M1)*** ***(A1)*** ***(M1)*** ***(A1)*(ft)** ***(M0)(A0)*(ft)**

Certain answers obtained from the GDC are worth 2 marks and working will not be seen. In these cases only one mark should be lost for accuracy.

eg, Chi-squared, correlation coefficient, mean

Markscheme	Candidates' Scripts	Marking
Chi-squared	(a) 7.7	***(G2)***
$7.68\ (7.67543\ldots)$ ***(A2)***	(b) 7.67	***(G1)***
	(c) 7.6	***(G1)***
	(d) 8	***(G0)***
	(e) 7	***(G0)***
	(e) 7.66	***(G0)***

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Regression line

Markscheme	Candidates' Scripts	Marking
$y = 0.888x + 13.5$ ***(A2)*** $(y = 0.887686\ldots x + 13.4895\ldots)$ If an answer is not in the form of an equation award at most ***(A1)(A0)***.	(a) $y = 0.89x + 13$	***(G2)*** *(both accepted)*
	(b) $y = 0.88x + 13$	***(G1)*** *(one rounding error)*
	(c) $y = 0.88x + 14$	***(G1)*** *(rounding error repeated)*
	(d) (i) $y = 0.9x + 13$ (ii) $y = 0.8x + 13$	***(G1)*** *(1 sf not accepted)*
	(e) $0.88x + 13$	***(G0)*** *(one rounding error and not an equation)*

Maximum/minimum/points of intersection

Markscheme	Candidates' Scripts	Marking
$(2.06, 4.49)$ ***(A1)(A1)*** $(2.06020\ldots, 4.49253\ldots)$	(a) $(2.1, 4.5)$	***(A1)(A1)*** *(both accepted)*
	(b) $(2.0, 4.4)$	***(A1)*** *(same rounding error twice)*
	(c) $(2.06, 4.4)$	***(A1)*** *(one rounding error)*
	(d) $(2, 4.4)$	***(A0)*** *(1sf not accepted, one rounding error)*

Rounding of an exact answer to 3 significant figures **should be accepted if performed correctly**. Exact answers such as $\frac{1}{4}$ can be written as decimals to fewer than 3 significant figures if the result is still exact. Reduction of a fraction to its lowest terms is **not** essential, however where an answer simplifies to an integer this is expected. Fractions that include a decimal in the numerator and/or the denominator are acceptable for showing correct substitution, but not as a final answer.

Ratios of π and answers taking the form of square roots of integers or any rational power of an integer (*eg,* $\sqrt{13}, 2^{\frac{2}{3}}, \sqrt[4]{5}$,) may be accepted as exact answers. All other powers (*eg,* of non-integers) and values of transcendental functions such as sine and cosine must be evaluated.

If the level of accuracy is specified in the question, a mark will be allocated for giving the answer to the required accuracy. In **all** such cases the final mark is not awarded if the rounding does not follow the instructions given in the question. A mark for specified accuracy can be regarded as a **(ft)** mark regardless of an immediately preceding ***(M0)***.

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6 Level of accuracy in finance questions

The accuracy level required for answers will be specified in all questions involving money. This will usually be either whole units or two decimal places. The first answer not given to the specified level of accuracy will not be awarded the final ***A*** mark. The markscheme will give clear instructions to ensure that only one mark per paper can be lost for incorrect accuracy in a financial question.

Example: A financial question demands accuracy correct to 2 dp.

Markscheme	Candidates' Scripts	Marking
\$231.62 (231.6189) ***(A1)***	(i) 231.6	***(A0)***
	(ii) 232	***(A0)*** *(Correct rounding to incorrect level)*
	(iii) 231.61	***(A0)***
	(iv) 232.00	***(A0)*** *(Parts (iii) and (iv) are both incorrect rounding to correct level)*

7 Units in answers

There will be specific questions for which the units are required and this will be indicated clearly in the markscheme. The first correct answer with no units or incorrect units will not be awarded the final ***A*** mark. The markscheme will give clear instructions to ensure that only one or two mark per paper can be lost for lack of units or incorrect units.
The units are considered only when the numerical answer is awarded ***(A1)*** under the accuracy rules given in Section 5.

Example:

Markscheme	Candidates' Scripts	Marking
(a) $37000\ \text{m}^2$ ***(A1)***	(a) $36000\ \text{m}^2$	***(A0)*** *(Incorrect answer so units not considered)*
(b) $3200\ \text{m}^3$ ***(A1)***	(b) $3200\ \text{m}^2$	***(A0)*** *(Incorrect units)*

If no method is shown and the answer is correct but with incorrect or missing units award G marks with a one mark penalty.

8 Graphic Display Calculators

Candidates will often be obtaining solutions directly from their calculators. They must use mathematical notation, not calculator notation. No method marks can be awarded for incorrect answers supported only by calculator notation. The comment 'I used my GDC' cannot receive a method mark.

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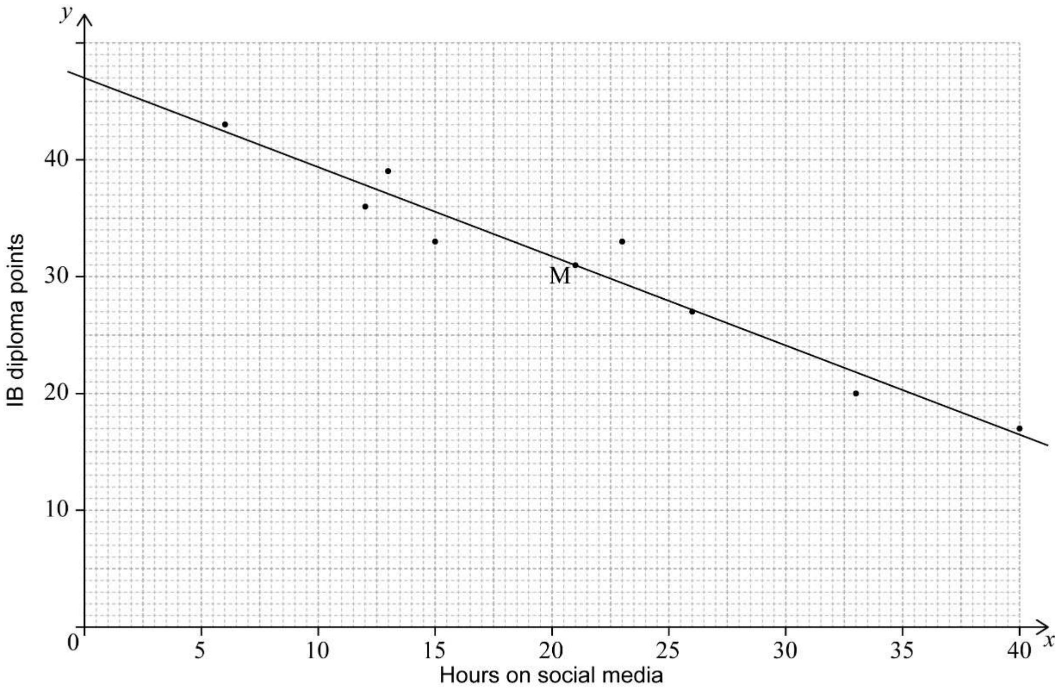
1. (a)



(A4)

Notes: Award ***(A1)*** for correct scale and labelled axes.
Award ***(A3)*** for 7 or 8 points correctly plotted,
(A2) for 5 or 6 points correctly plotted,
(A1) for 3 or 4 points correctly plotted.
Award at most ***(A0)(A3)*** if axes reversed.
Accept x and y sufficient for labelling.
If graph paper is not used, award ***(A0)***.
If an inconsistent scale is used, award ***(A0)***. Candidates' points should be read from this scale **where possible** and awarded accordingly.
A scale which is too small to be meaningful (ie mm instead of cm) earns ***(A0)*** for plotted points.

[4 marks]

(b) (i) $\bar{x} = 21$ ***(A1)***

(ii) $\bar{y} = 31$ ***(A1)***

[2 marks]

(c) $(\bar{x}, \bar{y})$ correctly plotted on graph ***(A1)*(ft)**
this point labelled M ***(A1)***

Note: Follow through from parts (b)(i) and (b)(ii).
Only accept M for labelling.

[2 marks]

continued…

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Question 1 continued

(d) -0.973 $(-0.973388\ldots)$ ***(G2)***

Note: Award ***(G1)*** for 0.973, without minus sign.

[2 marks]

(e) $y = -0.761x + 47.0$ $(y = -0.760638\ldots x + 46.9734\ldots)$ ***(A1)(A1)(G2)***

Notes: Award ***(A1)*** for $-0.761x$ and ***(A1)*** $+47.0$. Award a maximum of ***(A1)(A0)*** if answer is not an equation.

[2 marks]

(f) line on graph ***(A1)*(ft)*(A1)*(ft)**

Notes: Award ***(A1)*(ft)** for **straight line** that passes through their M, ***(A1)*(ft)** for line (extrapolated if necessary) that passes through $(0, 47.0)$. If M is not plotted or labelled, follow through from part (e).

[2 marks]

(g) $y = -\frac{2}{3}(34) + \frac{125}{3}$ ***(M1)***

Note: Award ***(M1)*** for correct substitution.

19 (points) ***(A1)(G2)***

[2 marks]

(h) extrapolation ***(R1)***

OR

34 hours is outside the given range of data ***(R1)***

[1 mark]

Note: Do not accept ‘outlier’.

Total [17 marks]

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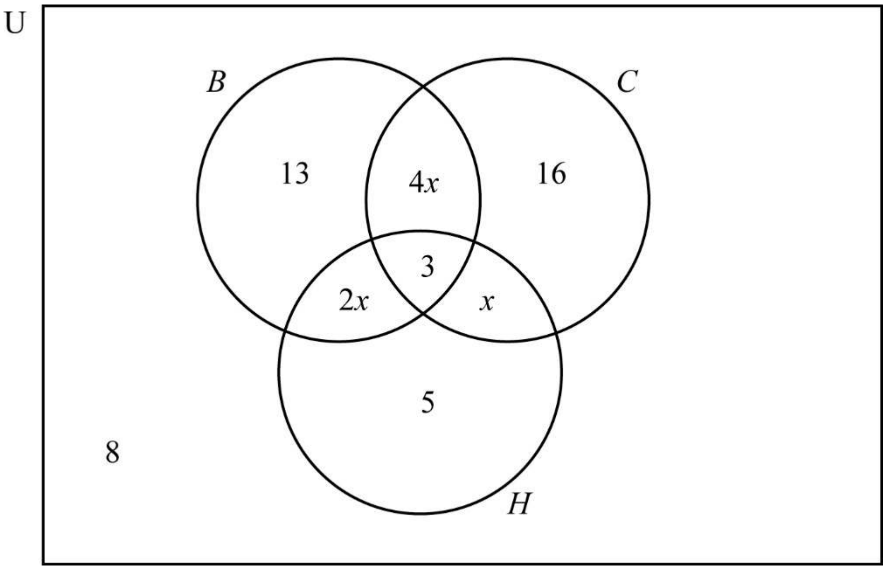
2. (a)



(A5)

Notes: Award ***(A1)*** for rectangle and three labelled intersecting circles (U need not be seen),
(A1) for 3 in the correct region,
(A1) for 8 in the correct region,
(A1) for 5, 13 and 16 in the correct regions,
(A1) for x, $2x$ and $4x$ in the correct regions.

[5 marks]

(b) $8+13+16+3+5+x+2x+4x=66$ ***(M1)***

Note: Award ***(M1)*** for **either** a completely correct equation **or** adding all the terms from **their** diagram in part (a) and equating to 66.
Award ***(M0)(A0)*** if their equation has no x.

$7x = 66-45$ **OR** $7x+45=66$ ***(A1)***

Note: Award ***(A1)*** for adding their like terms correctly, **but only** when the solution to their equation is equal to 3 and is consistent with their original equation.

$x=3$ ***(AG)***

Note: The conclusion $x=3$ must be seen for the ***(A1)*** to be awarded.

[2 marks]

(c) 15 ***(A1)*(ft)**

Note: Follow through from part (a). The answer must be an integer.

[1 mark]

continued…

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Question 2 continued

(d) (i) $\frac{42}{66}\left(\frac{7}{11}, 0.636, 63.6\%\right)$ ***(A1)*(ft)*(A1)(G2)***

> **Note**: Award ***(A1)*(ft)** for numerator, ***(A1)*** for denominator. Follow through from their Venn diagram.

(ii) $\frac{3}{9}\left(\frac{1}{3}, 0.333, 33.3\%\right)$ ***(A1)(A1)*(ft)*(G2)***

> **Note**: Award ***(A1)*** for numerator, ***(A1)*(ft)** for denominator. Follow through from their Venn diagram.

[4 marks]

Total [12 marks]

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3. (a) $2 \times 4 - 1 - 7 = 0$ (or equivalent) ***(R1)***

Note: For ***(R1)*** accept substitution of $x = 1$ or $y = 4$ into the equation followed by a confirmation that $y = 4$ or $x = 1$.

(since the point satisfies the equation of the line,) A lies on L_1 ***(A1)***

Note: Do not award ***(A1)(R0)***.

[2 marks]

(b) $\frac{1+5}{2}$ **OR** $\frac{4+12}{2}$ seen ***(M1)***

Note: Award ***(M1)*** for at least one correct substitution into the midpoint formula.

$(3, 8)$ ***(A1)(G2)***

Notes: Accept $x = 3$, $y = 8$. Award ***(M1)(A0)*** for $\left(\frac{1+5}{2}, \frac{4+12}{2}\right)$. Award ***(G1)*** for each correct coordinate seen without working.

[2 marks]

(c) $\sqrt{(5-1)^2 + (12-4)^2}$ ***(M1)***

Note: Award ***(M1)*** for a correct substitution into distance between two points formula.

$= 8.94 \left(4\sqrt{5}, \sqrt{80}, 8.94427\ldots\right)$ ***(A1)(G2)***

[2 marks]

(d) gradient of $\text{AC} = \frac{12-4}{5-1}$ ***(M1)***

Note: Award ***(M1)*** for correct substitution into gradient formula.

$= 2$ ***(A1)***

Note: Award ***(M1)(A1)*** for gradient of $\text{AC} = 2$ with or without working

gradient of the normal $= -\frac{1}{2}$ ***(M1)***

Note: Award ***(M1)*** for the negative reciprocal of their gradient of AC.

continued…

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Question 3 continued

$y-8=-\frac{1}{2}(x-3)$ **OR** $8=-\frac{1}{2}(3)+c$ ***(M1)***

Note: Award ***(M1)*** for substitution of their point and gradient into straight line formula. This ***(M1)*** can **only** be awarded where $-\frac{1}{2}$ (gradient) is correctly determined as the gradient of the normal to AC.

$2y-16=-(x-3)$ **OR** $-2y+16=x-3$ **OR** $2y=-x+19$ ***(A1)***

Note: Award ***(A1)*** for correctly removing fractions, **but only** if their equation is equivalent to the given equation.

$2y+x-19=0$ ***(AG)***

Note: The conclusion $2y+x-19=0$ must be seen for the ***(A1)*** to be awarded. Where the candidate has **shown** the gradient of the normal to $\text{AC}=-0.5$, award ***(M1)*** for $2(8)+3-19=0$ and ***(A1)*** for (therefore) $2y+x-19=0$. Simply substituting $(3, 8)$ into the equation of L_2 with no other prior working, earns no marks.

[5 marks]

(e) $(6, 6.5)$ ***(A1)(A1)(G2)***

Note: Award ***(A1)*** for 6, ***(A1)*** for 6.5. Award a maximum of ***(A1)(A0)*** if answers are not given as a coordinate pair. Accept $x=6,\ y=6.5$.
Award ***(M1)(A0)*** for an attempt to solve the two simultaneous equations $2y-x-7=0$ and $2y+x-19=0$ algebraically, leading to at least one incorrect or missing coordinate.

[2 marks]

(f) 3.3541 ***(A1)***

Note: Answer <u>must</u> be to 5 significant figures.

[1 mark]

continued…

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Question 3 continued

(g) $2\times\frac{1}{2}\times\sqrt{80}\times\frac{\sqrt{45}}{2}$ ***(M1)(M1)***

Notes: Award ***(M1)*** for correct substitution into area of triangle formula. If their triangle is a quarter of the rhombus then award ***(M1)*** for multiplying their triangle by 4. If their triangle is a half of the rhombus then award ***(M1)*** for multiplying their triangle by 2.

OR

$\frac{1}{2}\times\sqrt{80}\times\sqrt{45}$ ***(M1)(M1)***

Notes: Award ***(M1)*** for doubling MD to get the diagonal BD, ***(M1)*** for correct substitution into the area of a rhombus formula. Award ***(M1)(M1)*** for $\sqrt{80}\times$ their (f).

$=30$ ***(A1)*(ft)*(G3)***

Notes: Follow through from parts (c) and (f). $8.94\times3.3541=29.9856...$

[3 marks]

Total [17 marks]

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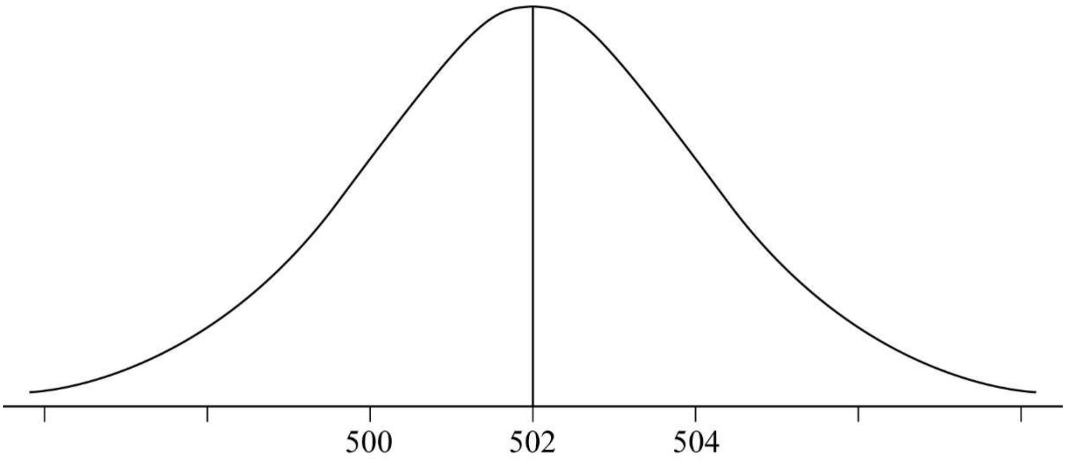
4. (a)



(A1)(A1)

Notes: Award ***(A1)*** for bell shape with mean of 502.
Award ***(A1)*** for an indication of standard deviation *eg* 500 and 504.

[2 marks]

(b) (i) $0.921 \ (0.920968\ldots, 92.0968\ldots\%)$ ***(G2)***

Note: Award ***(M1)*** for a diagram showing the correct shaded region.

(ii) $1500 \times 2 \times 0.920968\ldots$ ***(M1)***

$= (\$)\ 2760 \ (2762.90\ldots)$ ***(A1)*(ft)*(G2)***

Note: Follow through from their answer to part (b)(i).

[4 marks]

(c) $1500 \times 0.16 \times 0.079031\ldots$ ***(M1)***

Notes: Award ***(A1)*** for $1500 \times 0.16 \times$ their $(1 - 0.920968...)$.

OR

$(1500 - 1381.45) \times 0.16$ ***(M1)***

Notes: Award ***(M1)*** for $(1500 -$ their $1381.45) \times 0.16$.

$= (\$)19.0 \ (18.9676...)$ ***(A1)*(ft)*(G2)***

[2 marks]

(d) 347 (grams) (346.614…) ***(G3)***

Notes: Award ***(G2)*** for an answer that rounds to 346.
Award ***(G1)*** for 353.385… seen without working (for finding the top 3 %).

[3 marks]

Total [11 marks]

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5. (a) $(\text{BD} =)\ \sqrt{95^2 + 40^2}$ ***(M1)***

> **Note**: Award ***(M1)*** for correct substitution into Pythagoras' theorem.

$= 103\ (\text{m})\ \left(103.077\ldots,\ 25\sqrt{17}\right)$ ***(A1)(G2)***

[2 marks]

(b) $\cos \text{B}\hat{\text{A}}\text{D} = \dfrac{105^2 + 70^2 - (103.077\ldots)^2}{2 \times 105 \times 70}$ ***(M1)(A1)*****(ft)**

> **Note**: Award ***(M1)*** for substitution into cosine rule, ***(A1)*****(ft)** for their correct substitutions. Follow through from part (a).

$(\text{B}\hat{\text{A}}\text{D}) = 68.9^\circ\quad (68.8663\ldots)$ ***(A1)*****(ft)*****(G2)***

> **Note**: If their 103 used, the answer is $68.7995\ldots$

[3 marks]

(c) (Area of $\text{ABD} =$) $\dfrac{1}{2} \times 105 \times 70 \times \sin(68.8663\ldots)$ ***(M1)(A1)*****(ft)**

> **Notes**: Award ***(M1)*** for substitution into the trig form of the area of a triangle formula.
> Award ***(A1)*****(ft)** for their correct substitutions.
> Follow through from part (b).
> If 68.8° is used the area $= 3426.28\ldots\ \text{m}^2$.

$= 3430\,\text{m}^2\quad (3427.82\ldots)$ ***(A1)*****(ft)*****(G2)***

[3 marks]

(d) area of $\text{ABCD} = \dfrac{1}{2} \times 40 \times 95 + 3427.82\ldots$ ***(M1)***

> **Note**: Award ***(M1)*** for correctly substituted area of triangle formula **added** to their answer to part (c)

$= 5330\,\text{m}^2\quad (5327.83\ldots)$ ***(A1)*****(ft)*****(G2)***

[2 marks]

continued…

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Question 5 continued

(e) $\frac{1}{2}\times 105\times \text{AP}\times \sin(68.8663\ldots) = 0.5\times 5327.82\ldots$ ***(M1)(M1)***

Notes: Award ***(M1)*** for the correct substitution into triangle formula. Award ***(M1)*** for equating their triangle area to half their part (d).

$(\text{AP} =)\ 54.4\ (\text{m})\quad (54.4000\ldots)$ ***(A1)*****(ft)*****(G2)***

Notes: Follow through from parts (b) and (d).

[3 marks]

(f) $\text{BP}^2 = 105^2 + (54.4000\ldots)^2 - 2\times 105\times (54.4000\ldots)\times \cos(68.8663\ldots)$ ***(M1)(A1)*****(ft)**

Notes: Award ***(M1)*** for substituted cosine rule formula. Award ***(A1)*****(ft)** for their correct substitutions. Accept the exact fraction $\frac{53}{147}$ in place of $\cos(68.8663\ldots)$. Follow through from parts (b) and (e).

$(\text{BP} =)\ 99.3\ (\text{m})\quad (99.3252\ldots)$ ***(A1)*****(ft)*****(G2)***

Notes: If 54.4 and $\cos(68.9)$ are used the answer is $99.3567\ldots$

[3 marks]

Total [16 marks]

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6. (a) $(A =)\ \pi r^2 + 2\pi rh$ ***(A1)(A1)***

> **Note**: Award ***(A1)*** for either πr^2 **OR** $2\pi rh$ seen. Award ***(A1)*** for two correct terms added together.

[2 marks]

(b) 500000 ***(A1)***

> **Notes**: Units **not** required.

[1 mark]

(c) $500000 = \pi r^2 h$ ***(A1)*****(ft)**

> **Notes**: Award ***(A1)*****(ft)** for equating $\pi r^2 h$ to their part (b).
> Do not accept $V = \pi r^2 h$ unless V is explicitly defined as their part (b).

[1 mark]

(d) $A = \pi r^2 + 2\pi r\left(\dfrac{500000}{\pi r^2}\right)$ ***(A1)*****(ft)*****(M1)***

> **Note**: Award ***(A1)*****(ft)** for their $\dfrac{500000}{\pi r^2}$ seen.
> Award ***(M1)*** for correctly substituting **only** $\dfrac{500000}{\pi r^2}$ into a **correct** part (a).
> Award ***(A1)*****(ft)*****(M1)*** for rearranging part (c) to $\pi rh = \dfrac{500000}{r}$ and substituting for πrh in expression for A.

$A = \pi r^2 + \dfrac{1000000}{r}$ ***(AG)***

> **Notes**: The conclusion, $A = \pi r^2 + \dfrac{1000000}{r}$, must be consistent with their working seen for the ***(A1)*** to be awarded.
> Accept 10^6 as equivalent to 1000000.

[2 marks]

(e) $2\pi r - \dfrac{1000000}{r^2}$ ***(A1)(A1)(A1)***

> **Note**: Award ***(A1)*** for $2\pi r$, ***(A1)*** for $\dfrac{1}{r^2}$ or r^{-2}, ***(A1)*** for -1000000.

[3 marks]

continued…

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Question 6 continued

(f) $2\pi r - \dfrac{1000\,000}{r^2} = 0$ ***(M1)***

Note: Award ***(M1)*** for equating their part (e) to zero.

$r^3 = \dfrac{1000\,000}{2\pi}$ **OR** $r = \sqrt[3]{\dfrac{1000\,000}{2\pi}}$ ***(M1)***

Note: Award ***(M1)*** for isolating r.

OR

sketch of derivative function ***(M1)***
with its zero indicated ***(M1)***

$(r =)\ 54.2\ (\text{cm})\quad (54.1926\ldots)$ ***(A1)*(ft)*(G2)***

[3 marks]

(g) $\pi(54.1926\ldots)^2 + \dfrac{1000\,000}{(54.1926\ldots)}$ ***(M1)***

Note: Award ***(M1)*** for correct substitution of their part (f) into the given equation.

$= 27\,700\ (\text{cm}^2)\quad (27\,679.0\ldots)$ ***(A1)*(ft)*(G2)***

[2 marks]

(h) $\dfrac{27679.0\ldots}{2000}$ ***(M1)***

Note: Award ***(M1)*** for dividing their part (g) by 2000.

$= 13.8395\ldots$ ***(A1)*(ft)**

Notes Follow through from part (g).

14 (cans) ***(A1)*(ft)*(G3)***

Notes: Final ***(A1)*** awarded for rounding up their $13.8395\ldots$ to the next integer.

[3 marks]

Total [17 marks]

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